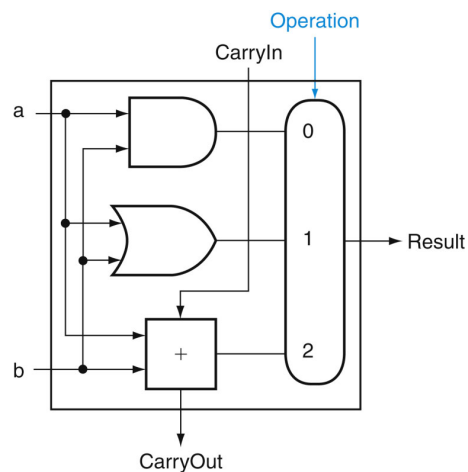


Chapter 3

ALU Design

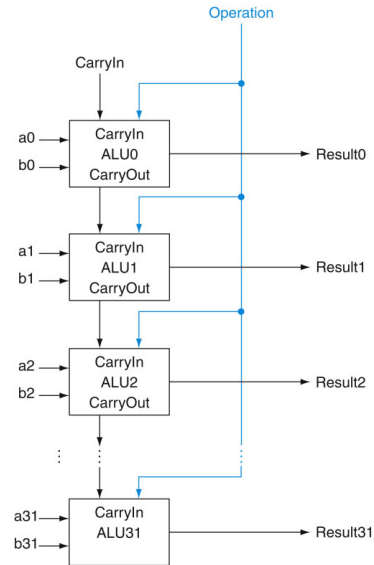
1-Bit ALU with Add, Or, And

- Multiplexor selects between Add, Or, And operations.



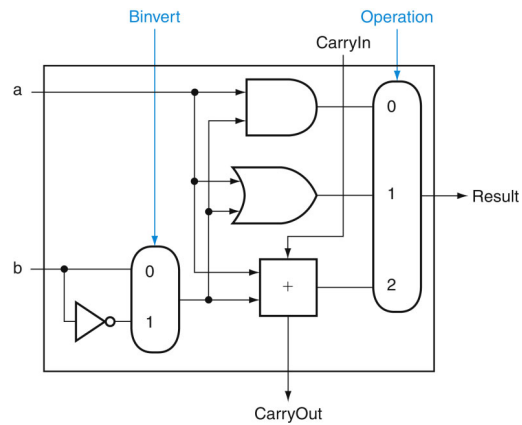
32-bit Ripple Carry Adder

- 1-bit ALUs are connected “in series” with the carry-out of 1 box going into the carry-in of the next box.

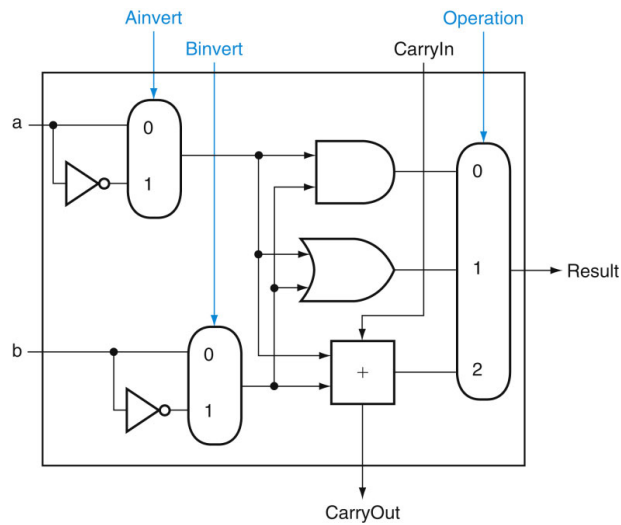


Incorporating Subtraction

- Must invert bits of B and add a 1 Include an inverter.
- CarryIn for the first bit is 1.
- The CarryIn signal (for the first bit) can be the same as the Binvert signal.

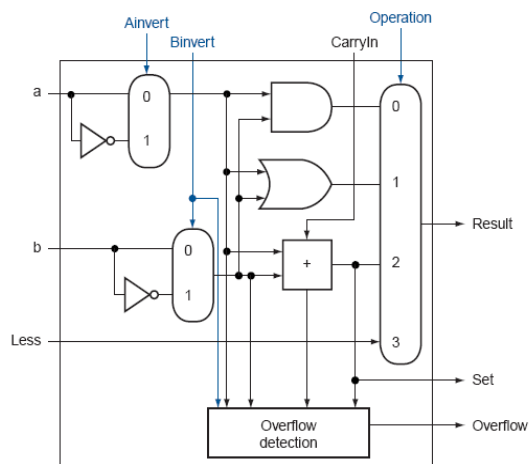


Incorporating NOR and NAND



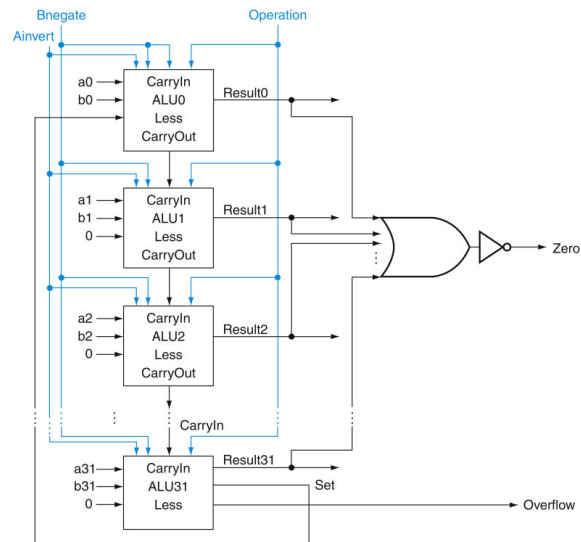
Incorporating SLT

- Perform $a - b$ and check the sign.
- New signal (Less) that is zero for ALU boxes 1-31.
- The 31st box has a unit to detect overflow and sign – the sign bit serves as the Less signal for the 0th box.

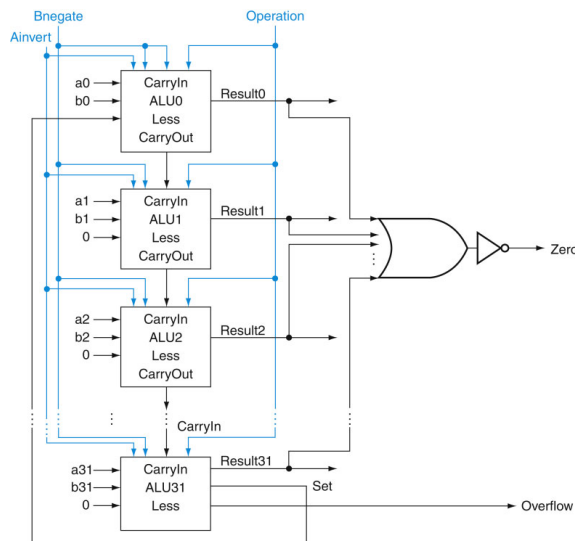


Incorporating BEQ

- Perform $a - b$ and confirm that the result is all zero's.



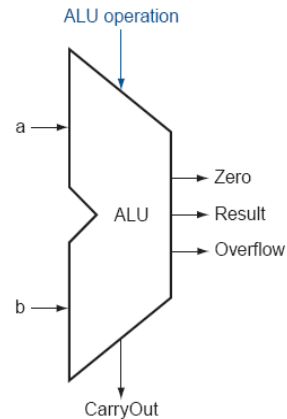
Control Lines



Control Lines

- What are the values of the control lines and what operations do they correspond to?

	Ai	Bn	Op
AND	0	0	00
OR	0	0	01
Add	0	0	10
Sub	0	1	10
SLT	0	1	11
NOR	1	1	00



Speed of Ripple Carry

- The carry propagates through every 1-bit box: each 1-bit box sequentially implements AND and OR – total delay is the time to go through 64 gates!
- You know that any logic equation can be expressed as the sum of products – so it should be possible to compute the result by going through only 2 gates!
- Caveat: need many parallel gates and each gate may have a very large number of inputs – it is difficult to efficiently build such large gates, so we'll find a compromise:
 - Moderate number of gates.
 - Moderate number of inputs to each gate.
 - Moderate number of sequential gates traversed.

Computing CarryOut

$$\begin{aligned}\text{CarryIn1} &= b_0.\text{CarryIn0} + a_0.\text{CarryIn0} + a_0.b_0 \\ \text{CarryIn2} &= b_1.\text{CarryIn1} + a_1.\text{CarryIn1} + a_1.b_1 \\ &= b_1.b_0.c_0 + b_1.a_0.c_0 + b_1.a_0.b_0 + \\ &\quad a_1.b_0.c_0 + a_1.a_0.c_0 + a_1.a_0.b_0 + a_1.b_1\end{aligned}$$

...

CarryIn32 = a really large sum of really large products.

- Potentially fast implementation as the result is computed by going thru just 2 levels of logic – unfortunately, each gate is enormous and slow.

Generate and Propagate

Equation re-phrased:

$$\begin{aligned}C_{i+1} &= a_i.b_i + a_i.C_i + b_i.C_i \\ &= (a_i.b_i) + (a_i + b_i).C_i\end{aligned}$$

Stated verbally, the current pair of bits will *generate* a carry if they are both 1 and the current pair of bits will *propagate* a carry if either is 1

$$\begin{aligned}\text{Generate signal} &= a_i.b_i \\ \text{Propagate signal} &= a_i + b_i\end{aligned}$$

Therefore, $C_{i+1} = G_i + P_i . C_i$

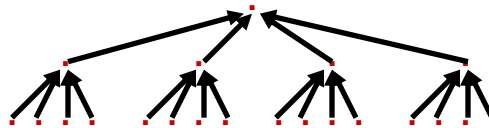
Generate and Propagate

$$\begin{aligned}c_1 &= g_0 + p_0.c_0 \\c_2 &= g_1 + p_1.c_1 \\&= g_1 + p_1.g_0 + p_1.p_0.c_0 \\c_3 &= g_2 + p_2.g_1 + p_2.p_1.g_0 + p_2.p_1.p_0.c_0 \\c_4 &= g_3 + p_3.g_2 + p_3.p_2.g_1 + p_3.p_2.p_1.g_0 + p_3.p_2.p_1.p_0.c_0\end{aligned}$$

Either,
a carry was just generated, or
a carry was generated in the last step and was propagated, or
a carry was generated two steps back and was propagated by both
the next two stages, or
a carry was generated N steps back and was propagated by every
single one of the N next stages

Divide and Conquer

- The equations on the previous slide are still difficult to implement as logic functions – for the 32nd bit, we must AND every single propagate bit to determine what becomes of c_0 (among other things).
- Hence, the bits are broken into groups (of 4) and each group computes its group-generate and group-propagate.
- For example, to add 32 numbers, you can partition the task as a tree.



P and G for 4-bit Blocks

- Compute P0 and G0 (super-propagate and super-generate) for the first group of 4 bits (and similarly for other groups of 4 bits)
 $P0 = p0.p1.p2.p3$
 $G0 = g3 + g2.p3 + g1.p2.p3 + g0.p1.p2.p3$
- Carry out of the first group of 4 bits is
 $C1 = G0 + P0.c0$
 $C2 = G1 + P1.G0 + P1.P0.c0$
...
- By having a tree of sub-computations, each AND, OR gate has few inputs and logic signals have to travel through a modest set of gates (equal to the height of the tree).

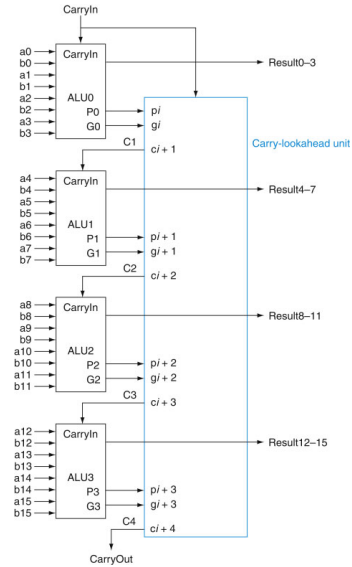
Example

Add	A	0001	1010	0011	0011
and	B	1110	0101	1110	1011
	g	0000	0000	0010	0011
	p	1111	1111	1111	1011
	P	1	1	1	0
	G	0	0	1	0

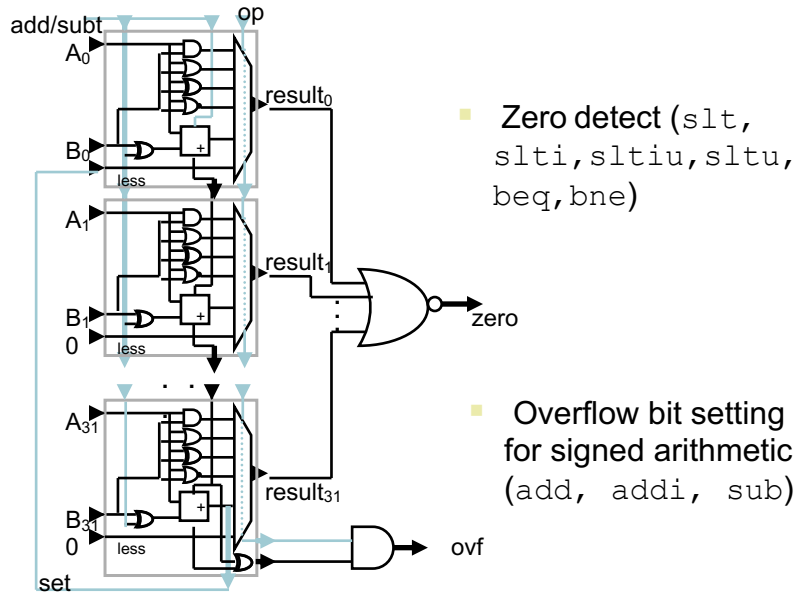
C4 = 1

Carry Look-Ahead Adder

- 16-bit Ripple-carry takes 32 steps.
- This design takes how many steps?



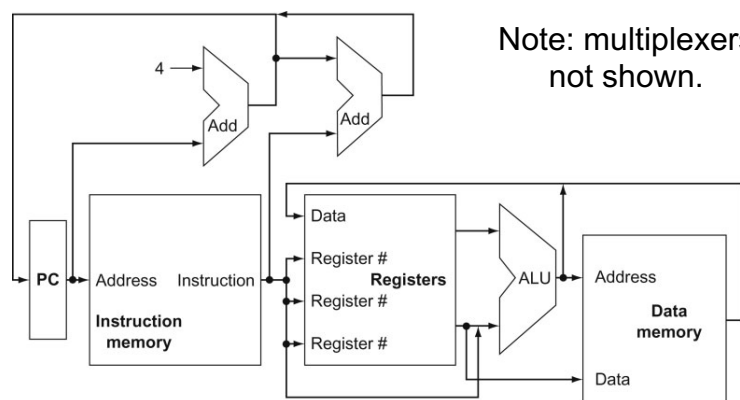
Review - A MIPS ALU Implementation



Implementation Overview

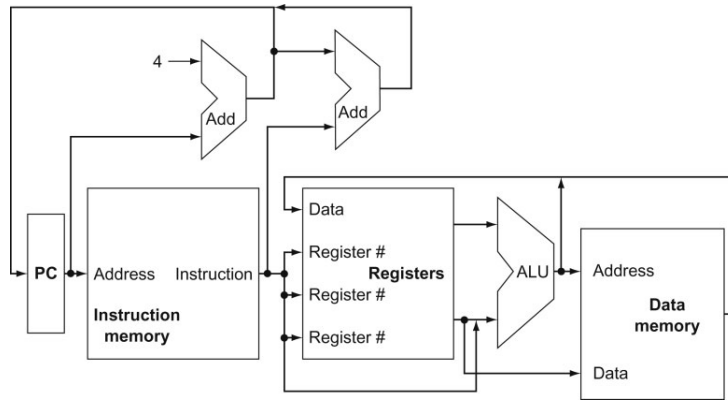
- We need memory
 - to store instructions
 - to store data
 - for now, let's make them separate units
- We need registers, ALU, and a whole lot of control logic
- CPU operations common to all instructions:
 - use the program counter (PC) to pull instruction out of instruction memory
 - read register values

Big Picture



- What is the role of the Add units?
- Explain the inputs to the data memory unit.
- Explain the inputs to the ALU.
- Explain the inputs to the register unit.

Clocking Methodology



- Which of the above units need a clock?
- What is being saved (latched) on the rising edge of the clock? Keep in mind that the latched value remains there for an entire cycle.